

# Multipath Rejection Through Spatial Processing

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## **BIOGRAPHY**

Alison Brown is the President and CEO of NAVSYS Corporation. She has a PhD in Mechanics, Aerospace, and Nuclear Engineering from UCLA, an MS in Aeronautics and Astronautics from MIT, and an MA in Engineering from Cambridge University. In 1986 she founded NAVSYS Corporation. Currently she is a member of the GPS-III Independent Review Team and Scientific Advisory Board for the USAF and serves on the GPS World editorial advisory board.

## **ABSTRACT**

The development of low cost, small size antenna arrays and digital antenna electronics has resulted in small, affordable GPS phased arrays and digital receivers that are suitable for commercial applications. NAVSYS High-gain Advanced GPS Receiver (HAGR) product includes the capability to spatially process signals from up to 16-antenna elements. Previous papers have presented the HAGR's capability to perform beamforming, in both static and mobile environments, providing over 10 dB of additional signal gain on each satellite tracked. This capability results in highly accurate code and carrier tracking providing superior performance for high accuracy applications, such as kinematic GPS.

In this paper, a further enhancement to the HAGR digital signal processing is described. A spatial signal processing algorithm has been developed that allows multipath signals to be detected and an adaptive antenna array pattern is then used to minimize their received signal power while still applying gain in the direction of the direct GPS signals. Test data is included showing the benefit of this method in reducing the multipath errors on both the pseudo-range and carrier-phase observations.

## **INTRODUCTION**

Multipath errors are caused by the receiver tracking a composite of the direct GPS signals and GPS signals

reflected from nearby objects. The resulting phase error is a function of the phase offset between the direct and multipath signals and the relative signal strength. For a fixed installation, these errors appear as biases, changing only as the line-of-sight to the satellite changes due to the satellite motion. In a mobile application, the multipath errors will also change due to the motion of the antenna.

GPS receivers commonly use broad antenna gain patterns so that all of the satellites above the horizon can be tracked. Unfortunately, this increases the susceptibility of GPS receivers to multipath reflections of the GPS signals from nearby objects. Multipath signals from below the receiver antenna can be removed using antennas with good low elevation signal rejection, such as a choke ring, but signals arriving from elevations above the antenna cannot be rejected without also eliminating the satellite signals that are needed to obtain high accuracy GPS solutions. If the GPS receiver cannot be located above all potential multipath sources, signals arriving from buildings or nearby vehicles can cause significant multipath errors that will not be rejected by the antenna pattern

Signal processing techniques have also previously been developed for reducing the effect of multipath errors on the tracking loops. These techniques include methods that apply temporal filtering of the multipath errors, multipath correction from site calibration or correlation shape correction using multiple digital correlators. While many of these methods have proven successful in reducing the effect of multipath on the code tracking loops, temporal filtering and multi-correlator signal processing have no effect on the multipath carrier phase errors. The spatial processing design described in this paper minimizes the multipath effect on both the code and carrier tracking loops within the GPS receiver.

Until recently, however, only military GPS receivers have taken advantage of spatial signal processing to improve the robustness of the GPS signal through the use of

Controlled Reception Pattern Antennas (CRPAs). NAVSYS pioneered the first commercial receiver to include spatial signal processing, the High-gain Advanced GPS Receiver (HAGR). This receiver uses digital spatial processing to combine the signals from up to 16 antenna elements, which allows over 10 dB of gain to be applied to each GPS satellite tracked, when operating in a beam-forming mode [1,2,3]. The reprogrammable digital spatial processing approach adopted by the HAGR also enables adaptive beam and null-forming to be applied which can be used to reduce the effect of multipath errors.

### GPS MULTIPATH ERRORS

Multipath errors are caused by the receiver tracking a composite of the direct GPS signals and reflected GPS signals from nearby objects such as the ground or nearby buildings.

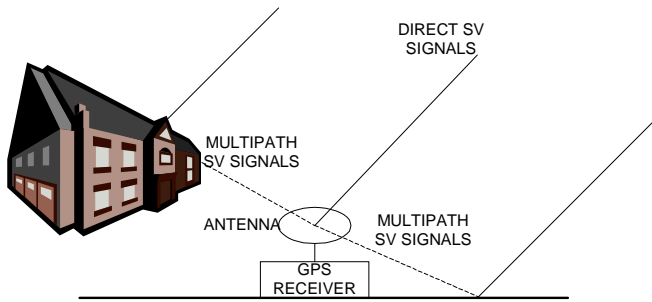


Figure 1 GPS Multipath Errors

The resulting phase error is a function of the phase offset between the direct and multipath signals and the relative signal strength. For a fixed installation, these errors appear as biases, changing only as the line-of-sight to the satellite changes due to the satellite motion.

$$s(t) = AC(t + \tau) \sin(\omega t + \theta) + A_M C(t + \tau_M) \sin(\omega t + \theta_M)$$

$$s(t)\hat{s}(t) = AR(\tau - \hat{\tau}) \sin(\omega t + \theta) + A_M R(\tau_M - \hat{\tau}) \sin(\omega t + \theta_M)$$

$$= (A + \tilde{A}) \sin(\omega t + \theta + \tilde{\theta})$$

$A$  is the amplitude of the direct signal and  $A_M$  is the amplitude of the reflected multipath signal  
 $\tau$  is the code phase offset for the direct signal and  $\tau_M$  is the code phase offset for the multipath signal  
 $\theta$  is the carrier phase offset for the direct signal and  $\theta_M$  is the carrier phase offset for the multipath signal

The above equation can be solved for the pseudo-range error that will be observed with the DLL tracking loops and the phase error that will be observed by the PLL tracking loops in a GPS receiver. This simplifies to the following expressions, if it is assumed that the multipath

reflections are relatively close to the antenna (compared with the C/A code chip length of 293 m) and that  $R(\tau)$  from the DLL is approximately equal to one for both the direct and multipath signals.

$$\tilde{\tau} = \frac{A_M^2}{A^2} \tau_M$$

This value gives the approximate maximum code phase error for a 1-chip Early/Late correlator assuming that the multipath delays are less than 1/2 chip. Below, the carrier phase error that is caused by the multipath errors is derived as a function of the received multipath signal strength and the carrier phase offset from the direct path.

$$\tilde{A} = |A + A_M e^{j\Delta\theta}| - A$$

$$\tilde{\theta} = \angle(A + A_M e^{j\Delta\theta})$$

$$\Delta\theta = \theta - \theta_M$$

In Figure 2, the effect of the multipath signal on the C/N0 envelope is shown as a function of the multipath signal power computed using the above equation. In Figure 3, the multipath phase angle error envelope is shown for the cases when the multipath amplitude is between 3dB and 20dB below the signal  $A_M=A$  (0 dB), ( $A_M=A/\sqrt{2}$ ), and ( $A_M=A/10$ ). As the phase of the multipath signals change relative to the direct signal, the observed signal C/N0 and the carrier phase errors will oscillate between the error bounds shown in Figure 2 and Figure 3.

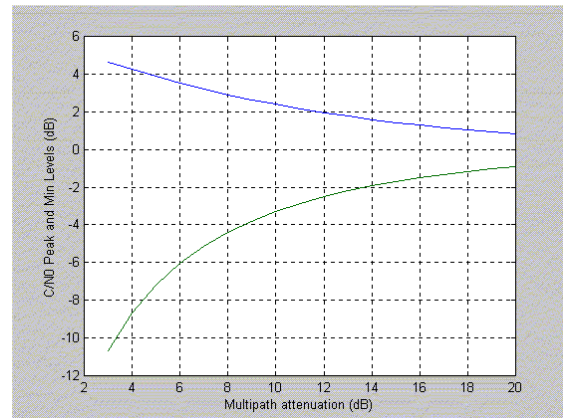
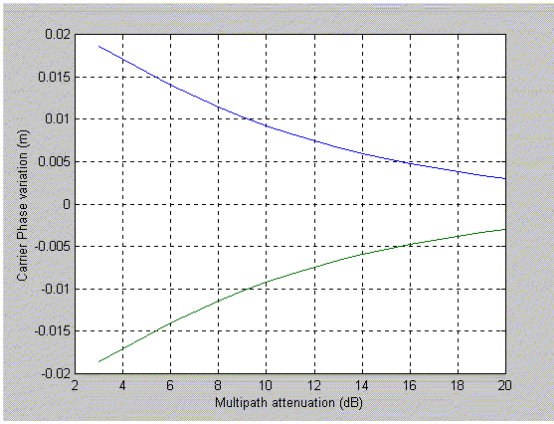


Figure 2 Multipath Amplitude Effect



**Figure 3 Multipath Phase Angle Error**

If the multipath signals are 10 dB below the direct signals, then the carrier phase errors will be below 1 cm. If the multipath signals can be maintained 20 dB below the direct signals, then the carrier phase errors will be below 3 mm.

### ADAPTIVE BEAM STEERING AND MULTIPATH NULLING ALGORITHM

To minimize the effect of the multipath signals, the digital antenna array is used to dynamically adjust the antenna array pattern to provide gain towards each of the satellites tracked, and also detect and place nulls in the direction of observed multipath signal sources. This approach has the following advantages over previous multipath reduction techniques employing fixed antenna patterns or temporal signal processing methods.

Array electronics estimates the presence of multipath sources. The multiple signals from the antenna elements are processed to dynamically estimate the presence of multipath. This provides a measure of the magnitude and effect of this error source that can be used to optimize siting of GPS reference stations to achieve high accuracy performance.

Array electronics adapts the antenna pattern to optimize the relative direct signal to multipath signal power. The beam steering is used to provide antenna gain to reinforce the direct signals and to apply nulling to minimize the effect of the received multipath signals.

A maximum likelihood algorithm was selected as our technical approach for implementing the beam-steering/multipath minimization algorithm. The digital signal output from each element in the GPS antenna array can be described by the following equation.

$$y_k(t) = \sum_{i=1}^{N_s} s_i(x_k, t) + n_k(t) + \sum_{j=1}^{N_M} s_{Mj}(x_k, t)$$

where  $s_i(x_k, t)$  is the  $i$ th GPS satellite signal received at the  $k$ th antenna element  
 $n_k(t)$  is the noise introduced by the  $k$ th DFE  
 $s_{Mj}(x_k, t)$  is the  $j$ th multipath signal received at the  $k$ th antenna element

The combined digital array signal,  $z(t)$ , is generated from summing the weighted individual digitized GPS signals. This can be expressed as the following equation.

$$z(t) = \underline{w}^T \underline{y}(t) = \underline{w}^T \left[ \sum_{i=1}^{N_s} s_i(t) \underline{e}_{si} + n_k(t) + \sum_{j=1}^{N_M} s_{Mj}(x_k, t) \right]$$

A parametric approach is used to estimate the set of multipath signal sources. This is computed based on the cross-correlation matrix computed from calibration signals ( $\underline{s}_c$ ) generated within the digital array logic that observes the residual signal on each of the antenna elements after the best estimate of the direct GPS signal has been removed.

$$P = E[(\underline{s}_c - \hat{A}_s \hat{\underline{e}}_s)(\underline{s}_c - \hat{A}_s \hat{\underline{e}}_s)^T]$$

$$\hat{\underline{1}}^{(B)} = (\hat{C}_B^N)^T \frac{\hat{\underline{x}}_U - \underline{x}_{SV}}{|\hat{\underline{x}}_U - \underline{x}_{SV}|}$$

$$\hat{\underline{e}}_s = \exp\left\{-\frac{2\pi}{\lambda} \hat{\underline{1}}^{(B)T} \underline{L}\right\}$$

The set of likely multipath signal sources are calculated as the set of values ( $\xi$ ) that minimize the following equation.

$$\min_{\xi} \|P - \hat{P}(\xi)\|^2$$

$$\hat{P}(\xi) = \left( \sum_{j=1}^{N_M} \hat{A}_{Mj} e^{j\hat{\theta}_{Mj}} \hat{\underline{e}}_{Mj} \right) \left( \sum_{j=1}^{N_M} \hat{A}_{Mj} e^{j\hat{\theta}_{Mj}} \hat{\underline{e}}_{Mj} \right)^T$$

The direction of the multipath signals is used to compute the optimal weights to apply beam-forming to the desired GPS satellite, and place a null on the estimated multipath signal directions. The weights required to minimize the antenna gain in the direction of the multipath signals and maximize the gain in the direction of the desired GPS satellites is then computed from the following equation.

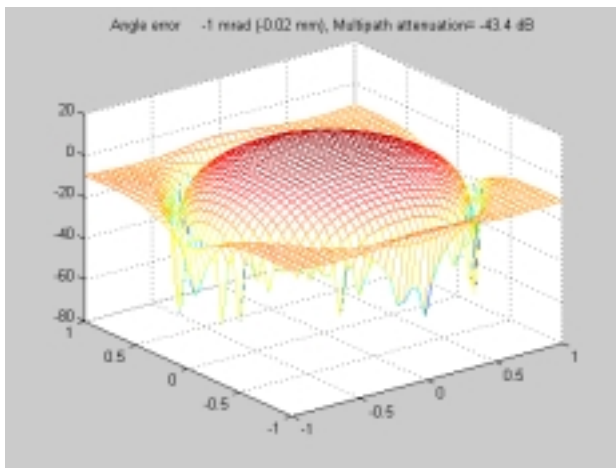
$$R = I\sigma_n^2 + \sum_{j=1}^{N_M} \hat{A}_{Mj}^2 \hat{\underline{e}}_{Mj} \hat{\underline{e}}_{Mj}^T$$

$$\underline{w} = \frac{R^{-1} \underline{e}_s}{\underline{e}_s^T R^{-1} \underline{e}_s}$$

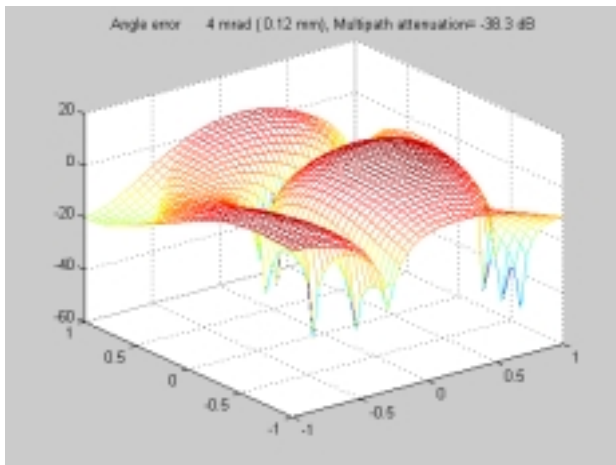
The signal-to-multipath gain improvement from the spatial processing after the multipath nulling weights have been applied can be computed from the following equation<sup>4</sup>.

$$S_i / M = \frac{\underline{w}^T \underline{e}_s \underline{e}_s^T \underline{w}}{\underline{w}^T \underline{e}_M \underline{e}_M^T \underline{w}}$$

The performance of the beam-former/multipath minimization algorithms is illustrated in Figure 4 and Figure 5 for simulated data assuming a single point-source multipath reflection at az/el [270,20]. These figures show that the multipath minimization algorithm applies over 40 dB of attenuation on the multipath signal in each of these cases. Moreover, the gain from the beam forming increases the C/No by 8 dB of gain on each satellite tracked. These results show the ideal situation with a “clean” multipath source. In the real-world environment multiple sources will be present which will reduce the null-depth possible.



**Figure 4 Single Multipath Source - SV at 90 deg elevation**



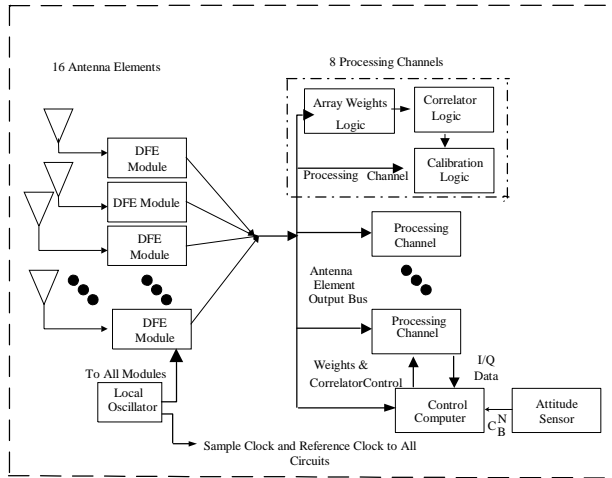
**Figure 5 Single Multipath Source - SV at 30 deg elevation**

## DIGITAL BEAMFORMING GPS RECEIVER (HAGR)

The GPS spatial processing algorithm for multipath nulling has been developed for implementation in NAVSYS’ High-gain Advanced GPS Receiver (HAGR)<sup>5</sup>. The reprogrammable digital spatial processing capability inherent in the HAGR allows GPS signals to be combined from as many as 16 antennas and create a multi-beam antenna pattern to apply gain to up to eight GPS satellites simultaneously. Currently a Precise Position System (PPS) version of the HAGR (the HAGR-200) is also in development with 12 channel L1 and L2 capability. The 16-element antenna array is shown in Figure 6. A small 7-element antenna configuration can also be used which leverages NAVSYS’ miniature antenna array (Mini-Array) technology<sup>6</sup>. Test data showing the beam-steering performance of the Mini-Array and HAGR in a mobile environment using the Mini-Array is included in reference [1].

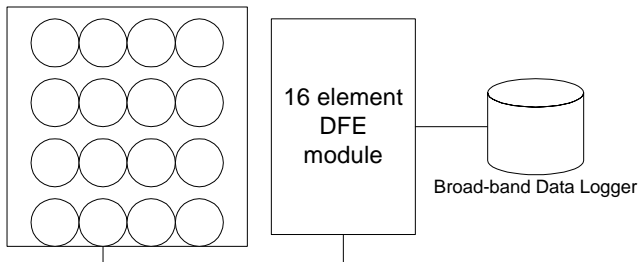


**Figure 6 HAGR 16-element antenna array**



**Figure 7 HAGR System Block Diagram**

The HAGR system architecture is shown in Figure 7. The signal from each antenna element is digitized using a Digital Front-End (DFE) as shown in Figure 8. The bank of digital signals is then processed by the HAGR digital-beam-steering card to create the composite digital beam-steered signal input for each of the receiver channels. The weights for each channel are dynamically downloaded through software control. These weights are adaptively computed in the HAGR host computer using the correlation results generated in the calibration logic to estimate in real-time the direction of multipath signal sources, as described in the previous section.



**Figure 8 Data Logger for 16 Element Antenna**

### MULTIPATH MINIMIZATION TEST DATA

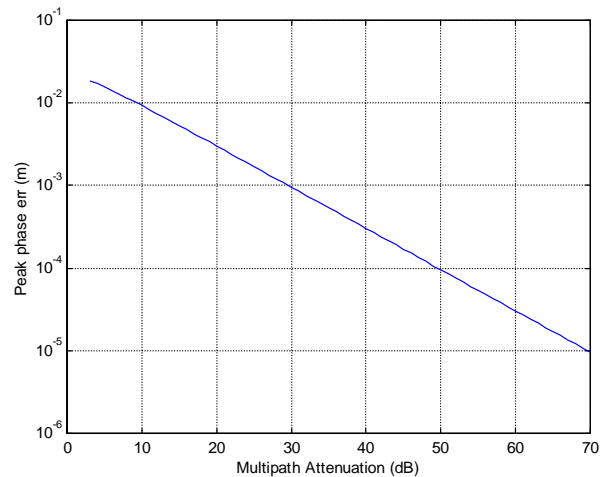
Test data was collected with our GPS digital storage receiver (DSR) to analyze the performance of the multipath minimization algorithms. This records the broad-band GPS sampled signals from each of 16 antenna elements at a rate of 40 Msps (Figure 8). The data was then played back through the HAGR signal processing to evaluate the effect of the spatial processing algorithms on reducing the code and carrier multipath errors. Since the DSR data is recorded at a rate of 80 Mbytes/sec, only a short window of data was collected for post-test analysis. The multipath detection and minimization algorithms were computed using a 20 msec sample of the data. The

accuracy of the multipath detection and null depth would be expected to improve when processing longer windows of data in the HAGR software.

In Figure 10 to Figure 13 the post-processed data is plotted showing the detected signal amplitude on each individual elements, and from the phased array solution combining the elements in the array. This shows that there is significant variation in the detected amplitude level (up to 15 dB) indicating the presence of strong multipath signals. In Figure 14 to Figure 21 the detected direct signal and multipath signals are plotted from the array. As expected, there are multiple multipath sources evident. In these figures, the antenna gain pattern is also plotted which shows that around 30-40 dB attenuation is placed in the direction of these multiple sources. The null depth in the direction of the jammer will also improve when using more than 20 msec of data in the estimation process.

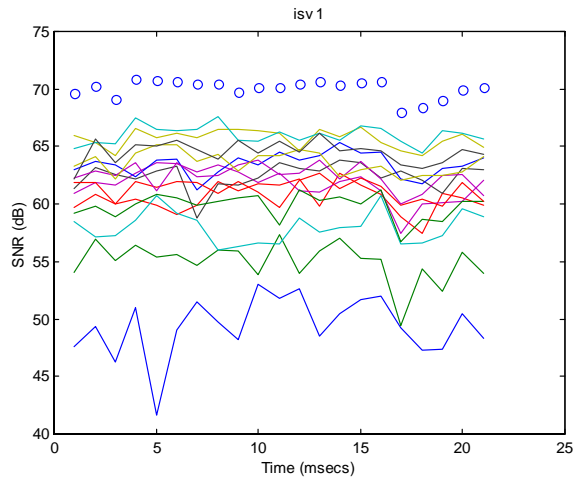
The spatial processing will reduce the multipath errors on both the code and carrier tracking loops. In Figure 9 the peak phase error that can be experienced from multipath is plotted as a function of multipath/signal strength. Since the multipath signal strength is always below the direct signal, even before the spatial processing has been applied, the additional attenuation provided by the spatial processing will reduce the multipath effect on the phase error to below 1 mm.

We are currently working on implementing the multipath detection and minimization algorithms in the HAGR software to allow real-time operation.

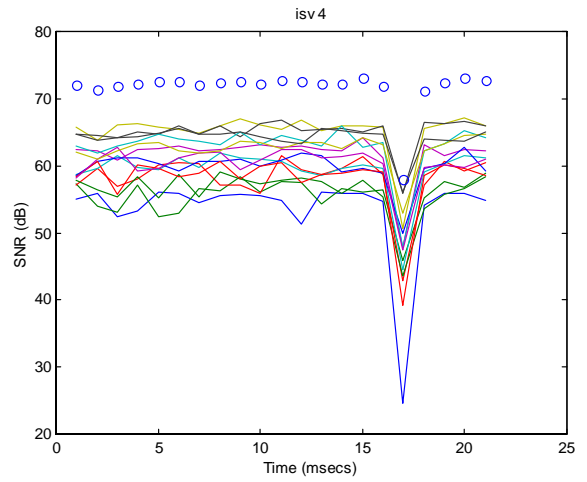


**Figure 9 Multipath Peak Phase error vs Attenuation (dB)**

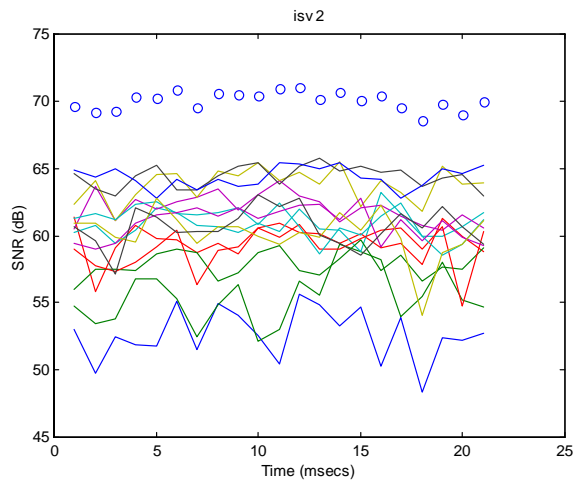




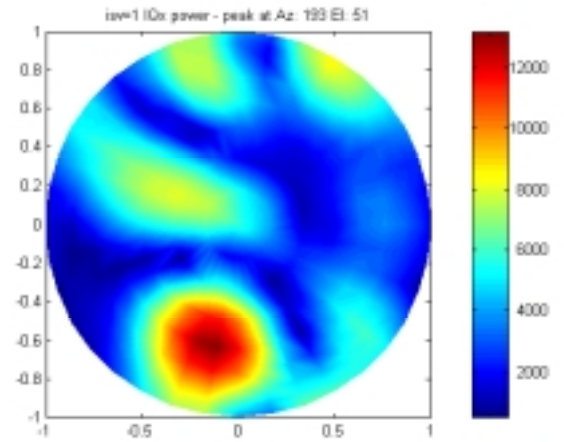
**Figure 10 SV 6 Amplitude from each element and array**



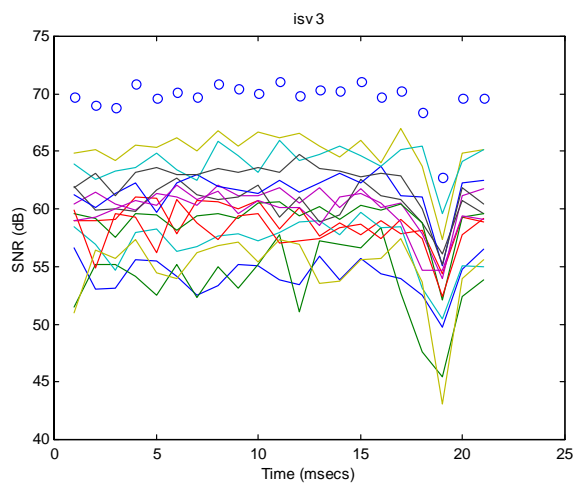
**Figure 13 SV 26 Amplitude from each element and array**



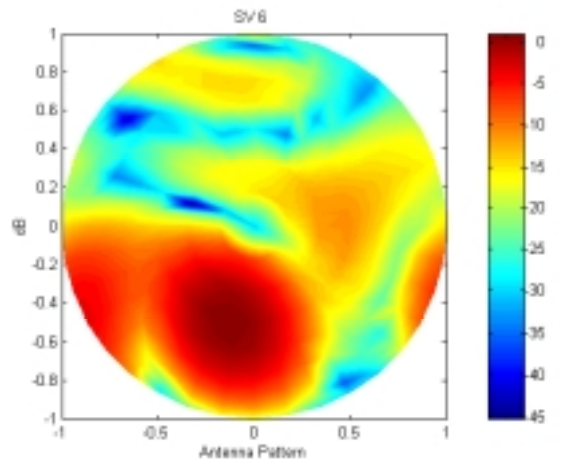
**Figure 11 SV 17 Amplitude from each element and array**



**Figure 14 SV 6 Detected Direct and Multipath Signal Power**



**Figure 12 SV 23 Amplitude from each element and array**



**Figure 15 SV 6 Multipath Nulling Antenna Pattern**

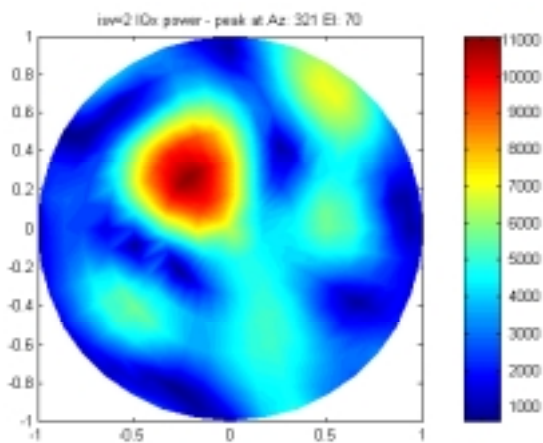


Figure 16 SV 17 Detected Direct and Multipath Signal Power

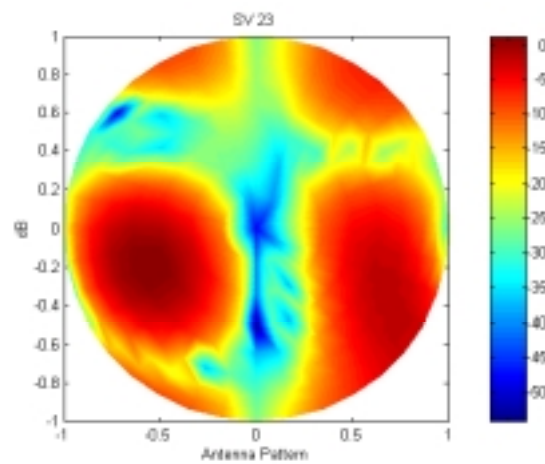


Figure 19 SV 23 Multipath Nulling Antenna Pattern

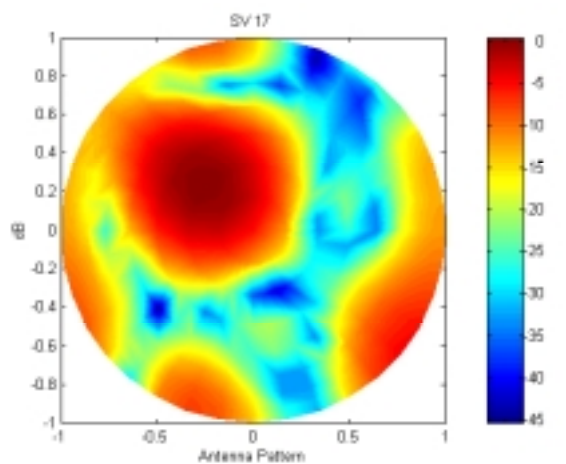


Figure 17 SV 17 Multipath Nulling Antenna Pattern

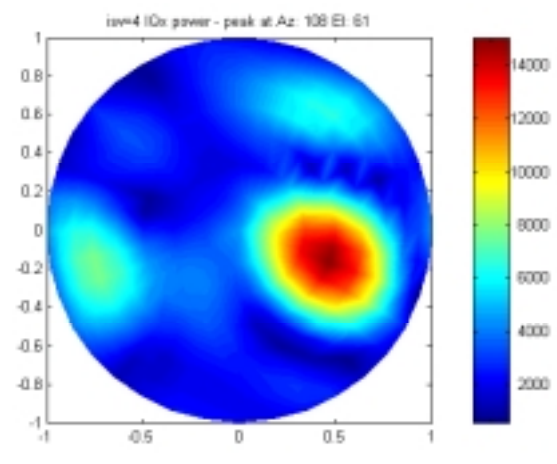


Figure 20 SV 26 Detected Direct and Multipath Signal Power

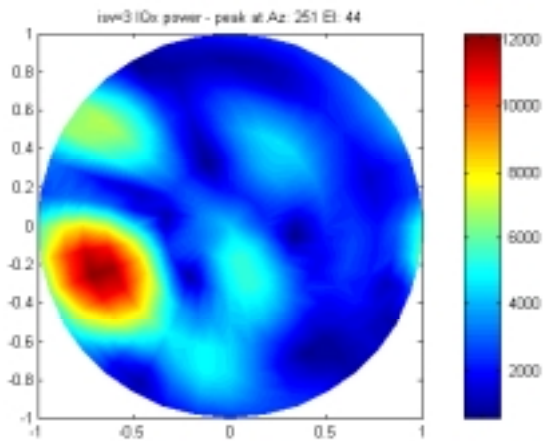


Figure 18 SV 23 Detected Direct and Multipath Signal Power

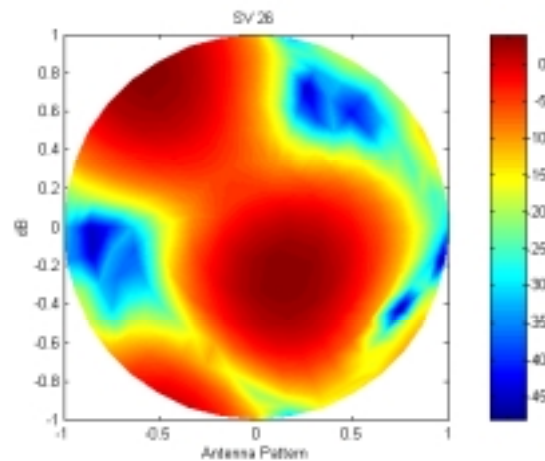


Figure 21 SV 26 Multipath Nulling Antenna Pattern

### CONCLUSION

The analysis and test data collected have demonstrated that digital spatial processing can be used to detect GPS multipath signals and minimize their effect by applying

gain towards the GPS satellites (beam-steering) and nulls in the direction of the detected GPS multipath signals. Since the digital spatial processing has the effect of reducing the multipath/signal amplitude level before the correlation process is performed, this method of multipath rejection improves the tracking performance on both the code and carrier tracking loops.

The dynamic beam/null-steering spatial processing capability described in this paper (patent pending) is being implemented in our HAGR commercial products. This technology offers benefits to high accuracy GPS applications that rely on precise code and carrier phase observations. These applications include providing GPS reference data, kinematic GPS positioning and precise time transfer.

## REFERENCES

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- <sup>2</sup> A. Brown, J. Wang, "[High Accuracy Kinematic GPS Performance using a Digital Beam-Steering Array](#)," Proceedings of ION GPS-99, Nashville, Tennessee, September 1999.
- <sup>3</sup> A. Brown, H. Tseng and R. Kurtz, "[Test Results Of A Digital Beamforming GPS Receiver For Mobile Applications](#)," Proceedings of ION National Technical Meeting, Anaheim, California, January 2000
- <sup>4</sup> D. Johnson and D. Dudgeon, "Array Signal Processing, Concepts and Techniques", Prentice-Hall, 1993 pp51
- <sup>5</sup> Dr. Alison Brown, Randy Silva, Gengsheng Zhang, "[Test Results of a High Gain Advanced GPS Receiver](#)," ION 55<sup>th</sup> Annual Meeting, Cambridge, Massachusetts, June 1999
- <sup>6</sup> A. Brown and H. Tseng, "[Miniaturized GPS Antenna Array and Test Results](#)", Proceedings of GNSS 2000, Edinburgh, Scotland, May 2000