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Extended Differential GPS

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ABSTRACT

The Radio Technical Commission for Maritime Services (RTCM) Special Committee (SC)-104 on Differential GPS has established a standard data format for providing differential GPS services. The satellite pseudorange corrections that are broadcast in the RTCM SC-104 differential GPS navigation message lump together the errors from a number of different sources. This results in a degradation of the differential navigation accuracy if the user is operating at a distance from the differential reference station. Generally, the user must be within 100 nmi of the reference station to benefit fully from the differential GPS corrections.

This paper presents an Extended Differential GPS concept that extends the range of operation of the differential GPS message. By using a network of differential GPS stations, one can compute a differential GPS message that provides corrections for the different components of the pseudorange error. This allows the differential GPS navigation accuracy to be extended to a range of 1000 nmi from the master differential station.

INTRODUCTION

The Extended Differential GPS concept is illustrated in Figure 1. With conventional differential GPS methods, a single differential reference station broadcasts differential GPS corrections to users in a local region. In the Extended Differential GPS concept, the differential reference stations are networked together to one or more master stations. The master differential station receives the differential GPS satellite error corrections from the individual monitor stations, then combines these to format a differential GPS message that will be valid over the extended range illustrated in Figure 1. The Extended Differential GPS message is then broadcast to users over the extended range through a satellite communications link. This approach provides a highly accurate differential GPS service over a large geographic area. In the illustration shown, the network is assumed to extend over the continental United States (CONUS) using two master stations, each with an individual satellite communications link. By linking differential networks together, continent-to-continent, the range could be extended even farther to encompass portions of the Pacific and Atlantic oceans.

The following sections include an analysis of the GPS errors that affect the accuracy of the differential GPS solution. Also included is a discussion of the additional models that need to be incorporated into the differential GPS message to extend the range of operation.

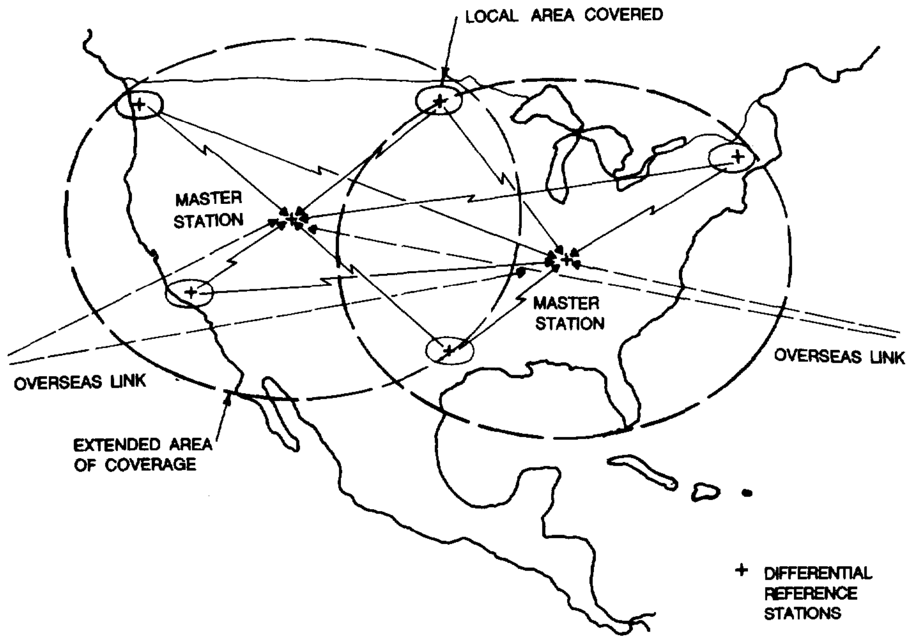

Fig. 1.—Extended Differential GPS Network

CONVENTIONAL DIFFERENTIAL GPS

With conventional differential GPS, a master station is used to track the GPS satellites, determine their range errors through comparison with a known reference solution, and broadcast differential GPS corrections to users in a local area. The accuracy of the final navigation solution is a function of the accuracy of the differential GPS satellite pseudorange error corrections.

The absolute navigation accuracy provided by GPS is a function of the accuracy of the pseudorange and delta-range measurements. The measurements are affected by the GPS system errors, such as ephemeris, clock, and selective availability (SA); atmospheric effects (ionosphere and troposphere); and receiver errors (noise, vehicle dynamics, and multipath). In some cases, differential GPS corrections can be used to reduce or eliminate these errors, resulting in improved performance. In the following paragraphs, the GPS absolute navigation error budget shown in Table 1 is discussed, and the error budget shown in Table 2 is derived for the differential GPS errors as a function of distance from the master station.

Clock and Ephemeris Errors

The errors allocated to the Space and Control Segments in Table 1 are 10 ft (1 σ) for the satellite clock timing error, and 9 ft (1 σ) for the error in the satellite ephemeris data. These errors are consistent with those specified for the GPS system in SS-GPS-300C [1].

The satellite clock timing error appears as a bias on the pseudorange measurement and so is totally removed by differential GPS corrections, as indicated

Table 1—GPS Absolute Navigation Error Budget

Source	Error Budget (ft)
Space Segment	
Clock Error	10
Control Segment	
Ephemeris Error	9
Selective Availability	90
Atmospheric Effects	
Ionospheric Delay	27
Tropospheric Delay	6
User Segment	
Receiver Noise	30
Multipath	10
User Equivalent Range Error (UERE) (RMS)	100
Navigation Accuracy 2 dRMS (HDOP = 1.5)	300

Table 2—Differential GPS Error Budget

Source/Range	0 nmi (ft)	100 nmi (ft)	500 nmi (ft)	1000 nmi (ft)	2000 nmi (ft)
Space Segment					
Clock Error	0	0	0	0	0
Control Segment					
Ephemeris Error	0	0.3	1.5	3	6
Selective Availability	0	0	0	0	0
Atmospheric Effects					
Ionospheric[1]	0	7.2	16	21	27
Tropospheric	0	6	6	6	6
Total (RMS)	0	9.4	17	22	28
User Segment					
Receiver Noise[2]	3	3	3	3	3
Multipath[3]	0	0	0	0	0
UERE (RMS)	3	9.8	17.4	22.2	28.5
Navigation Accuracy 2 dRMS (HDOP = 1.5)	9	30	52	66	86

Notes
[1]Ionospheric delay of 30 assumed.
[2]Receiver noise is filtered.
[3]Multipath rejection antenna used.

in the differential range error budget assigned in Table 2. However, the satellite position error does not cancel completely with differential navigation when the user and reference station are separated.

The satellite position error can be modeled as having along-track (ATK), cross-track (XTK), and radial (RAD) error vectors, as shown in Figure 2. Depending on the user's location, different components of these errors will subtend into the range from the user to the satellite. The satellite position errors will not, therefore, cancel completely unless the user and reference station are close to each other.

The contribution of the satellite position error components to the user range error (Pos_{err}) is given by equation (1). The angle α (RSO or USO) is between the line-of-sight vector from the reference station (**RS**) or user (**US**) to the satellite and the vertical from the earth's center to the satellite (**OS**). The angle β is between the along-track direction and the plane defined by the line-of-sight vector and the vertical to the satellite (ΔOSR or ΔOSU). The satellite position error vectors RAD, ATK, and XTK are in the radial, along-track, and cross-track directions, respectively. The relationship among the angle α, the radius of the satellite's orbit R_s, the radius of the user R_u, and the earth angle E (SOR or SOU) between the satellite and the user and reference station is given in equation (2).

$$Pos_{err} = \cos\alpha\ RAD + \sin\alpha\cos\beta\ ATK + \sin\alpha\sin\beta\ XTK \quad (1)$$

$$\sin\alpha = R_u \sin E / \sqrt{R_u^2 + R_s^2 - 2R_uR_s\cos E} \quad (2)$$

Experimental results have demonstrated typical satellite orbital errors of 3 ft in the radial direction (RAD), 21 ft along-track (ATK), and 10 ft cross-track (XTK) [2]. The pseudorange error, Pos_{err}, due to these satellite position errors can be computed using equation (1). Since the along-track error, ATK, is generally greatest, the maximum pseudorange error will occur when the line of sight to the user is in the along-track plane and the satellite is on the user's local horizon (zero elevation). It can be shown geometrically that this occurs

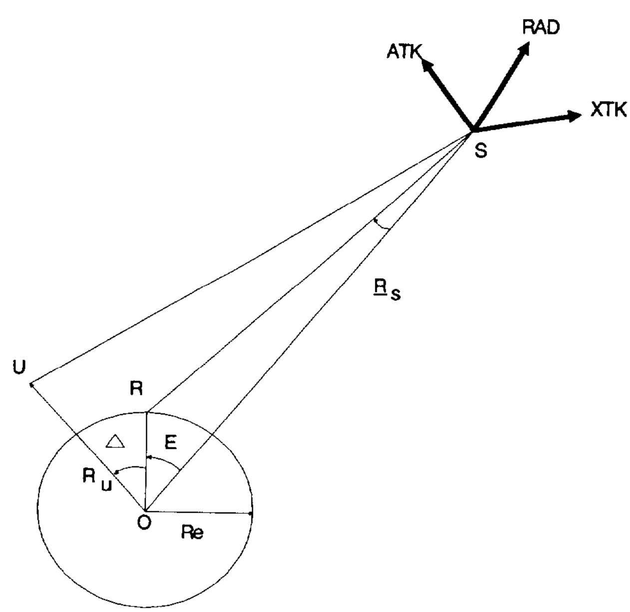

Fig. 2—GPS Position Errors

when the angle α satisfies the equation $\sin \alpha_m = R_e/R_s$, i.e., $\alpha_m = 13.9°$. Substituting this value for α into equation (1) with the errors RAD = 3 ft, ATK = 21 ft, and XTK = 10 ft results in a maximum pseudorange error, $Pos_{err,}$ of 8.5 ft. With these values for the ATK, XTK, and RAD satellite position errors, the pseudorange error, $Pos_{err,}$ will vary between 8.5 ft (at low elevation) and 3 ft (satellite directly overhead). These range errors are consistent with the 9 ft range error budget assigned in Table 1 to the satellite ephemeris errors.

If the GPS user and the differential reference station are separated by angle Δ, as shown in Figure 2, then the relative range error can be computed by substituting $E = E_0$ and $E = E_0 + \Delta$ into equation (2) and differencing the results of equation (1). The differential error is most sensitive to along-track (ATK) and cross-track (XTK) errors. In Figure 3, the differential error is shown as a fraction of the combined ATK and XTK errors in the plane of the line-of-sight vector and satellite radial vector. The error is shown for different angles, α (USO), between the line-of-sight vector and the satellite radial vector.

The differential error due to satellite position errors, $\Delta Pos_{err,}$ can be approximated using equation (3). This approximation is plotted in Figure 3, along with the exact differential errors. In equation (3), H is the vehicles' altitude, R_e is the earth's radius, R_s is the satellite's orbital radius, and Δ is the angular separation between the vehicle and the reference station in radians.

$$\Delta - Pos_{err}/\ (XTK \text{ or } ATK) = (R_e + H)\ \Delta\ /\ (R_s - R_e) \qquad (3)$$

Substituting the values $(R_e + H) = 3{,}444$ nmi and $R_s - R_e = 10{,}898$ nmi into equation (3) results in a differential error growth rate of approximately 10 percent of the ATK or XTK error per 1000 nmi separation. Figure 3 shows

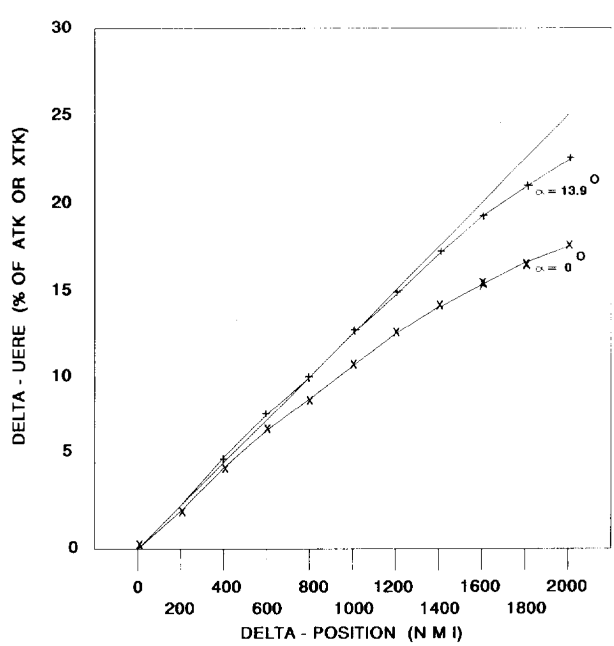

Fig. 3—Differential Errors Due to SV Position Errors

that this approximation is accurate to about 70 percent of the actual effect. If the along-track and cross-track errors result in a 25 ft error, the differential error will grow at approximately 3 ft/1000 nmi separation. In Table 2, the effect of this error is included in the differential error budget. It should be recognized that once the Standard Positioning Service (SPS) accuracy is degraded to 100 m (2 dRMS) (see the next section), the along-track and cross-track ephemeris errors may be significantly larger than 25 ft.

Selective Availability

The SA errors are introduced into the GPS signals on board the satellite to degrade the navigation accuracy provided by the SPS. Military user equipment includes a key that allows the SA errors to be removed from the pseudorange measurements. The SA errors are specified to degrade the horizontal navigation accuracy provided by the SPS to 100 m (2 dRMS). They can be modeled as a correlated noise process with standard deviation of 90 ft and a correlation time of a few minutes that acts as a slowly changing clock offset [3].

Since the SA errors act as a slowly changing bias on the pseudorange measurements, they can be completely removed using differential corrections. The only residual error remaining will be due to any time delay in applying the differential corrections. So long as the differential corrections are applied in near real time (within about 10 s), the residual error due to SA can be ignored.

Ionospheric Effects

The effect of the ionosphere on radio signals is proportional to the number of free electrons along the signal path. This value is known as the columnar electron density, N_c, or total electron content (TEC). N_c is computed from the integral of the electron density (electrons/m^3) along the signal path, and is generally given in total electron content units (TECU). One TECU is equivalent to 10^{16} electrons/m^2. If the signal path is in the vertical direction through the ionosphere, the columnar electron density is called the vertical electron content, N_{cv}.

The group delay introduced on the GPS satellite signals is proportional to the vertical TEC at the vehicle's location, scaled by an obliquity factor to account for slanted paths through the ionosphere for low-elevation satellites. The combined group delay ($c\Delta t$) is shown in equation (4), where f is the L1 frequency (1575.42 MHz) and El is the satellite's elevation angle.

$$
\begin{aligned}
c\Delta t &= \frac{40 \cdot 3}{f^2} N_{cv} \times Z(El)\ \text{m} = 0.53\ Z(El)\ \text{ft/TECU} \\
\text{where } Z(El) &= \sec\left[\sin^{-1}(0.94792 \cos El)\right]
\end{aligned}
\tag{4}
$$

The vertical electron content in the mid-latitude regions (e.g., CONUS) is typically around 50 TECU [4]. It can, on occasion, reach values as large as 100 TECU during periods of high solar activity, such as can be anticipated during the solar maximum years 1989–90. The ionospheric obliquity factor Z(El) given in equation (4) varies between a factor of 1 when the satellite is directly overhead (vertical) and a factor of 3 when the satellite is near the horizon. If the vertical TEC is 50 TECU, from equation (4), the ionospheric group delay will vary between 27 ft and 80 ft, depending on the satellite elevation angle.

An ionospheric model is broadcast by the GPS satellites to allow a single-frequency user to compensate for the ionospheric group delay. This model, described in [5] and [6], allows a user to compensate for approximately 50 percent RMS of the total delay. This will typically result in a residual ionospheric delay over CONUS of about 27 ft, as shown in Table 1.

The accuracy of the differential navigation solution is strongly dependent on the spatial decorrelation of the ionosphere. When the master station and vehicle are separated geographically, the satellite signals are passing through different areas of the ionosphere and so will experience different group delays. The variability of TEC as a function of distance has been studied in [7]. TEC data from two sets of stations were used: one aligned approximately along an east-west direction, the other approximately along a north-south direction. No significant difference in correlation distance was found with season. The fractional reduction of the ionospheric group delay, F(r), due to common errors canceling is related to the correlation coefficient, r, through the equation $F(r) = \sqrt{1-r^2}$.

The studies documented in [7] showed that ionospheric delays have a correlation coefficient of 0.7 over 2000 km in latitude and over 3000 km in longitude at northern mid-latitudes. It should be noted that this data was taken at common local times. The large diurnal change in the ionosphere will introduce a much larger variation over longitude for the data sets used in the differential GPS process as the data is taken at the same UTC time and so at different local times.

In Table 3, the percentage of the ionospheric delay remaining after differential GPS corrections have been applied is shown as a function of separation distance between stations. At 1000 nmi separation, the ionospheric spatial correlation coefficient is 0.7, which means that the ionospheric delay will be reduced to 71 percent of its original value by differential corrections. The GPS ionospheric model will reduce the ionospheric delay, on average, to 50 percent of the total value. Table 3 shows that differential GPS will improve the navigation performance over the GPS ionospheric model only when the separation distance between stations is less than 500 nmi.

The differential GPS group delay is shown in Figure 4 for a 30 ft and 90 ft group delay, respectively. The ionospheric group delay will typically lie between these values over CONUS, depending on the magnitude of the vertical TEC and the obliquity factor Z(El). Differential GPS will provide improved accuracy over the GPS ionospheric model (50 percent reduction) when the separation distance between the vehicles is less than 500 nmi. The residual ionospheric delay will be between 15 and 45 ft at this point. As will be discussed later, an updated ionospheric model can further reduce the ionospheric delay.

Table 3—Differential Ionospheric Delay Reduction

Distance (nmi)	0	1	10	50	100	500	1000	2000
F(r) (%)	0	2	8	17	24	52	71	91

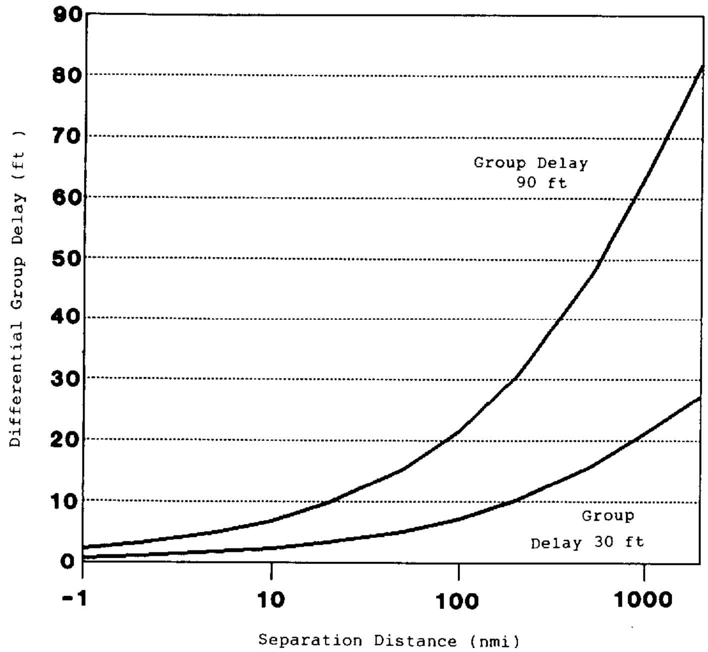

Fig. 4—Differential GPS Ionospheric Group Delay

Tropospheric Delay

The troposphere is the lower part of the atmosphere and is generally considered to extend up to an altitude of about 10 km. The tropospheric delay can typically be modeled using equation (5), where N_0 is the mean surface refractivity (3.2×10^{-4}), H is the vehicle's altitude, H_s is the tropospheric height (22,600 ft), and El is the satellite's elevation angle. The uncompensated delay can typically vary from 7 ft at high elevations to 80 ft at low elevations. The simple model given in equation (5) is adequate to reduce the residual tropospheric error to around 6 ft.

$$c\,\Delta t = N_0\,H_s\,e^{-H/H_s}\,\mathrm{cosec(El)} \tag{5}$$

Since the tropospheric delay is a function of local humidity, temperature, and altitude, differential GPS corrections are ineffective for compensating for tropospheric errors. The error budget allocated for the differential tropospheric errors in Table 2 is, therefore, the same as that allocated for absolute navigation in Table 1.

User Segment Errors

The user segment errors are introduced at the user's receiver and are, therefore, unaffected by differential GPS corrections. Error budgets are assigned in Tables 1 and 2 for the contribution of receiver noise and multipath errors to the user equivalent range error (UERE).

With a C/A-code receiver, the noise introduced by the code-tracking loops is generally on the order of 30 ft. This can be significantly reduced by the use of filtering techniques, such as a Kalman filter. For differential GPS, it is essential to reduce the receiver noise by filtering to improve the navigation accuracy. By using a Kalman filter, it is possible to reduce the effective receiver noise to within 3 ft (1 σ).

Multipath errors are introduced at the receiver from reflected GPS signals received by the antenna. These vary depending on the types of reflective surfaces around the vehicle (e.g., water, concrete, buildings) and the type of antenna used. Typically, a C/A-code receiver will experience multipath errors on the order of 10 ft. The effect of multipath can be reduced significantly through the use of an antenna that will reject reflected signals received from directions below the local horizon. With a specially designed antenna, it is possible to reduce the multipath errors to insignificant levels [8]. This error source is, therefore, ignored in the differential GPS error budget shown in Table 2.

EXTENDED DIFFERENTIAL GPS

Table 2 shows that a number of different errors limit the accuracy of the differential GPS solution as the distance from the reference station grows. The most significant error source is ionospheric delay, which dominates the error budget at distances of 100 nmi or greater from the reference station. The next largest error of concern is tropospheric delay, which introduces an error of 6 ft when using the simple correction model shown in equation (5). Once the range extends more than 1000 nmi from the reference station, satellite ephemeris errors also affect the accuracy of the differential GPS solution.

To extend the range of the differential GPS operation, the differential GPS message will need to include an ionospheric model, a tropospheric model, and an estimate of the satellite ephemeris errors. The following sections address the accuracy of available models, and ways of applying them to this problem.

Ionospheric Modeling

There exist a variety of ionospheric models that predict, with varying degrees of accuracy, the TEC in different parts of the world. The features of some of the most appropriate of these models are described in this section. The accuracy of predictions by all ionospheric models, however, is limited by the high temporal variability of the ionospheric delay.

Temporal Variability

The TEC exhibits both diurnal and periodic variations due to perturbations of the ionospheric F region from geomagnetic substorms, meteorological sources such as weather fronts, shock waves from supersonic aircraft, volcanic explosions, rocket launches, and other miscellaneous sources. While these short-term variations in TEC cover a large range of periods and amplitudes, common periods range from 20 to over 100 min, with amplitudes of a few percent of the background TEC.

Studies documented in [9] concluded that the useful prediction time for mid-latitude stations measuring the ionosphere was a function of local time, season, and long-term sunspot activity. However, in most cases there was no significant improvement over the use of monthly mean TEC predictions when actual data more than 3 h old was used. These studies also predicted the TEC data for the same geographic location and direction in the sky as their measurements. If the prediction is for a different location, the temporal correlation will be lower.

Because of the low temporal correlation of the ionosphere, a model must be updated in near real time (within at least 20 min) with current measurements of the ionospheric profile over the geographic region of interest to be effective in removing the group delay in the highly variable low-latitude region. This can be achieved with the extended differential GPS concept by making dual-frequency ionospheric measurements at the differential reference stations throughout the network shown in Figure 1. Dual-frequency P-code receivers may be used for this purpose, but are not required. Codeless receivers are now available which make measurements of the ionospheric delay by tracking the relative P-code phase between the L1 and L2 frequencies, without knowledge of the P-code. This provides a measurement of the ionospheric delay accurate to within 3 ft, with an uncertainty of the integer number of P-code cycles involved, ± 48 ft. Since the ionospheric delay can be estimated to within 48 ft using a rough model, this measurement is sufficient for accurately measuring the delay to each satellite. Codeless dual-frequency receivers are preferred for civil applications since they do not require periodic updates to be able to track the encrypted Y-code.

The dual-frequency ionospheric measurements from the reference stations will be transmitted to the differential master station along with the observed satellite range errors. At the master station, the measurements from the stations will be combined and used to update a real-time ionospheric model. A number of models that can be adapted for this purpose are described in the following sections.

GPS Ionospheric Model

To allow single-frequency C/A-code users to compensate partially for ionospheric group delay errors, an ionospheric model is broadcast by the GPS satellites [5,6]. This model uses eight coefficients to provide a correction for approximately 50 percent RMS of the total ionospheric range error. This algorithm is a severely truncated version of a much larger empirical model of TEC developed by Bent [10] over 13 years ago. The algorithm consists of a cosine representation of the diurnal curve, allowed to vary in amplitude and in period with user geomagnetic latitude. The algorithm coefficients were computed from an empirical model of worldwide ionospheric behavior, derived by Bent, for each 10 day period of the year and for several values of average solar flux conditions. Coefficients broadcast by the GPS satellites are updated once each 10 days, or sometimes more frequently if the 5 day running mean solar flux changes by a large amount during that period.

Because of the high degree of day-to-day variability in the TEC from the monthly mean value, the GPS model generally removes only 50 percent of the total delay. The eight coefficients are used to model the amplitude of the cosine term and its period as a function of geomagnetic latitude using a third-degree polynomial. The nighttime constant level term and the phase of the cosine curve are assumed constant.

Bent Model

The main purpose of the Bent model is to determine the TEC of the ionosphere to determine the group delay experienced by a radio signal. Documentation and a description of a computer program that implements this model are included in [10].

The parameters input to the Bent model are the date, universal time, locations of the transmitter and receiver, rate of change in elevation and altitude of the satellite or vehicle, operating frequency, solar flux (10.7 cm flux), and sunspot number. The Bent computer model [10] outputs the vertical electron content above the transmitter, the profile of vertical electron density with altitude, and the TEC along the path between the satellite and the ground. The model operates using an empirical database of 50,000 topside ionospheric soundings, 6,000 satellite measurements of electron density, and 400,000 bottomside soundings of the ionosphere. The data extends from 1962 to 1969 to cover the maximum and minimum of the solar cycle. The model can predict the ionospheric group delay at the mid-latitudes with an accuracy of 75–80 percent using the empirical database of ionospheric data.

The model also has the option to update the ionospheric profile using current data from local observation stations. Up to eight observations of the TEC separated by different amounts in time and space from the evaluation time and master station can be accepted. To obtain the best possible update, the observation times and stations should be the closest to the evaluation condition available and must be within 1000 nmi of the master site. Using current data, the accuracy of the model at mid-latitudes is improved to 90 percent of the total effect.

Differential GPS Ionospheric Model

To minimize the impact on the GPS receiver navigation software, a combination of the GPS ionospheric model and the updated Bent model is proposed for the extended differential message. The concept is illustrated in Figure 5. Ionospheric measurements from the differential reference stations to the different GPS satellites are used to update a computer implementation of the Bent model at the master station. Presently, the model allows for only eight measurement updates to be applied. This could be extended if it were demonstrated that this would improve the model accuracy. The updated Bent model provides a real-time model of the ionospheric group delay accurate to 10 percent of the total effect over a region of 1000 nmi from the central master station. A set of eight coefficients is then generated to provide a best fit to the model averaged over CONUS. The approximations made in the eight-coefficient GPS model reduce the accuracy of the final model by an estimated 10 percent. The updated eight-coefficient model is broadcast over the extended differential data link to users of the service.

By using the extended differential GPS ionospheric model instead of the model broadcast by the GPS satellites, single-frequency C/A-code users can remove approximately 80 percent of the ionospheric delay when they are within 1000 nmi of the master station. The extended differential GPS service should, therefore, be able to reduce the ionospheric errors to below 6 ft over a ±1000 nmi region by broadcasting an updated eight-coefficient GPS model. If a more sophis-

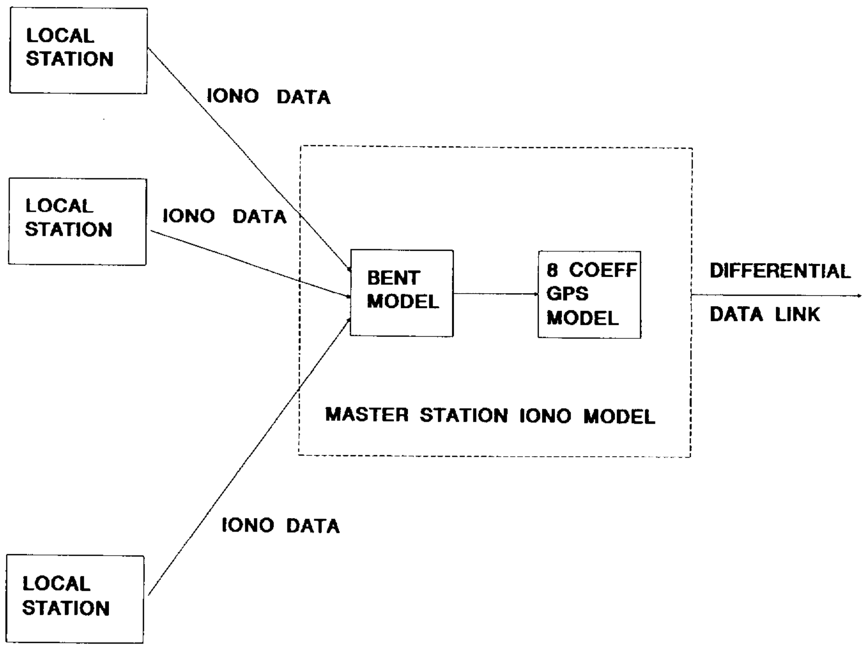

Fig. 5—Updated Ionospheric Model

ticated model were broadcast for the ionospheric delay and the differential GPS reference stations were placed every 600 nmi (10 deg) in latitude and longitude, the accuracy of the model and the range of operation could be further improved.

Tropospheric Modeling

An accurate, real-time tropospheric model can also be broadcast by the master station to reduce the differential GPS error from the residual tropospheric delay. A special-purpose model suitable for this purpose has already been designed by the Air Force Cambridge Research Laboratory (AFCRL) [11]. This model computes the tropospheric group delay as a function of surface refractivity, altitude, and elevation angle. The model can be updated in real time through a three-coefficient fit to the surface refractivity, N_s, averaged over latitude (L) and altitude (H). With measurements of the surface refractivity, the AFCRL tropospheric model is accurate to 4 percent of the total delay.

The extended differential tropospheric model can be updated in real time using surface refractivity measurements made over the region of coverage by the differential reference stations. The accuracy of the residual tropospheric delay using the extended differential GPS tropospheric model can, therefore, be expected to lie between 0.3 and 3 ft, depending on the elevation angle of the satellite.

Satellite Ephemeris Computation

The satellite ephemeris errors can also be observed by the extended differential GPS network using measurements from the reference stations. The reference stations over the region of coverage are observing the satellite range measurements. This data can be processed in a number of different ways to solve for the satellite ephemeris errors.

Networks of GPS monitor stations already exist which are used to compute precise ephemeris data for the GPS satellites through postprocessing. One such network is the Cooperative International GPS Network (CIGNET), which has tracking sites located at Mojave, Westford, Richmond, Yellowknife, Tromso, Onsala, Wettzel, Tsukaba, and Kokee Park, as illustrated in Figure 6. The accuracy of the precise ephemeris provided by CIGNET is about 1 part in 10^7, resulting in an estimated ephemeris error of 8 ft. It would be possible to implement a similar process in real time using the observed satellite range errors from the extended differential network to compute a set of precise orbital parameters. By broadcasting ATK, XTK, and RAD corrections for the ephemeris errors observed for each satellite, the orbital accuracy could be improved to within 8 ft. This would reduce the differential error introduced by the ephemeris data over ± 1000 nmi to below 2 ft.

It becomes significantly more complicated to compute the precise ephemeris data once SA is implemented. To be effective, the ephemeris determination program must be capable of separating out clock dither effects from errors in the broadcast ephemeris data. This can be achieved by using the extended differential GPS network to compute the satellite ephemeris data and SA clock errors using the method illustrated in Figure 7.

The range errors from four or more reference stations can be used to perform a least squares solution for position and time errors for each satellite, just as range measurements from four or more satellites to a user can solve for the user's position and time errors. The station range measurements observe the

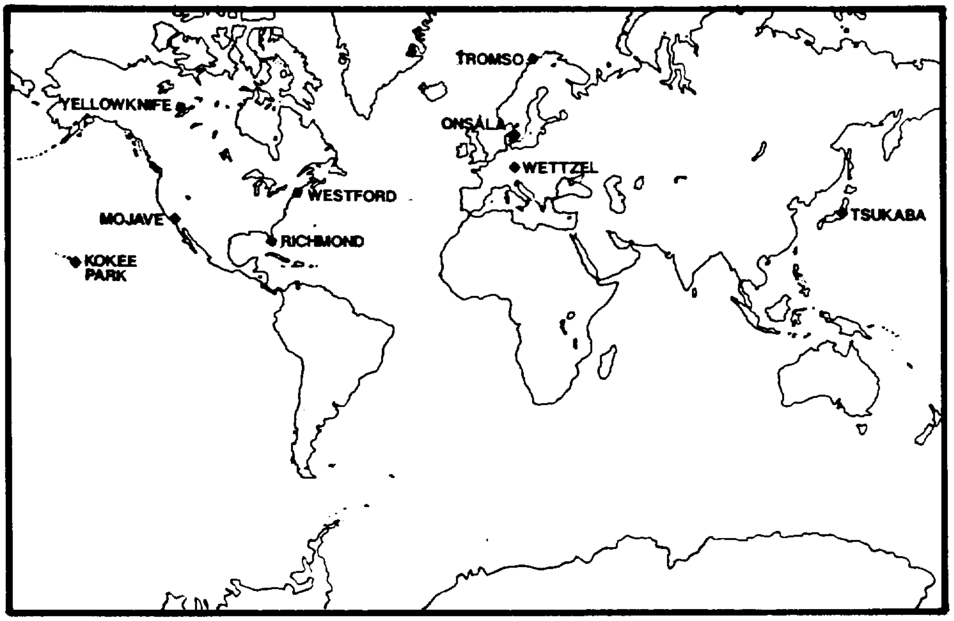

Fig. 6—CIGNET Network

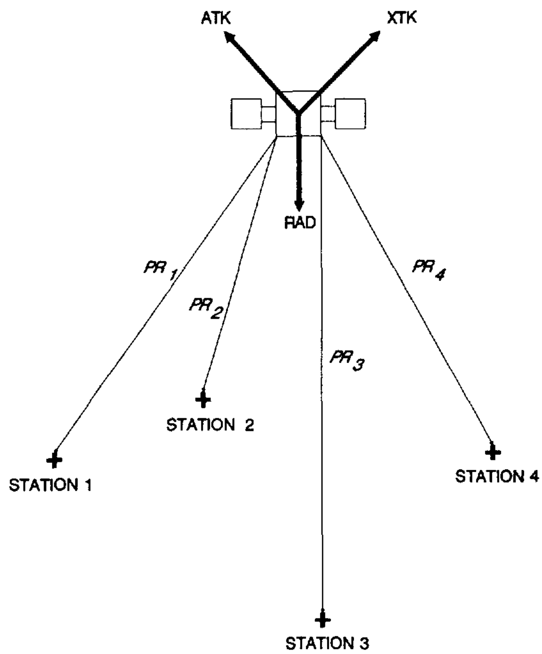

Fig. 7—Ephemeris Error Observation

satellite ATK, XTK, RAD, and clock errors (B_s) through the line-of-sight vectors from the reference stations to the satellite (x_i, y_i, z_i). The clock error term, B_s, will include both the clock offset in the satellite ephemeris data and the error introduced by SA. The least squares solution for the satellite errors using these measurements is shown in equation (7). The matrix H consists of the line-of-sight vectors from the differential stations to the satellite.

$$\begin{bmatrix} ATK \\ XTK \\ RAD \\ B_S \end{bmatrix} = \mathbf{GH}^T \begin{bmatrix} PR_1 \\ PR_2 \\ \cdot \\ \cdot \\ PR_n \end{bmatrix} \tag{7}$$

$$\text{where } \mathbf{G} = (\mathbf{H}^T\mathbf{H})^{-1} \text{ and } \mathbf{H} = \begin{bmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ x_n & y_n & z_n & 1 \end{bmatrix}$$

The accuracy of the final satellite position solution will be a function of the geometry provided by the ranges from the reference stations and the accuracy of the range error measurements. If the line-of-sight vectors from the satellite are defined in along-track (x), cross-track (y), and radial (z) directions, dilutions of precision can be computed for the accuracy of the ATK, XTK, RAD, and time/SA estimates, respectively. These dilutions of precision are defined as follows: ADOP = G_{11} is the along-track dilution of precision, XDOP = G_{22} is the cross-track dilution of precision, RDOP = G_{33} is the radial dilution of precision, and TDOP = G_{44} is the time/SA dilution of precision.

Since the differential GPS errors are most sensitive to the along-track and cross-track errors combined, a space vehicle horizontal dilution of precision term (SVHDOP) is defined which is the combination of the ADOP and XDOP terms.

$$SVHDOP^2 = ADOP^2 + XDOP^2 \tag{8}$$

If the range measurements to the satellites are accurate to 3 ft (1 σ), the combined along-track and cross-track residual errors will be given by SVHDOP $\times$ 3 ft after the satellite position corrections have been applied from the extended differential GPS message. The residual position error will result in a differential GPS range error of approximately 10 percent of the along-track or cross-track errors over 1000 nmi. The residual differential range error can, therefore, be approximated by equation (9).

$$\Delta\, Pos_{err} = SVHDOP \times 0.3 \text{ ft} / 1000 \text{ nmi} \tag{9}$$

The RDOP and TDOP terms represent how well the extended differential network can differentiate between SA (clock dither errors) and satellite position errors in the radial direction. This is not critical since the radial error differs from a clock error only as a $(1 - \cos\alpha)$ effect (see Figure 2). Over the region in which the satellite is visible, this causes a variation of 3 percent = $(1 - \cos 13.9°)$ between the SA errors and a radial satellite position error. Even if the satellite radial position error is as high as 30 ft, the variation in the UERE over the region of satellite visibility will be only 1 ft. The satellite radial position errors and clock errors, including SA, can therefore be lumped together without affecting the extended differential GPS navigation accuracy.

Performance Simulation

The solution geometry provided by the CIGNET stations in observing the satellite ephemeris was simulated for the 6-plane, 24-satellite GPS constellation. The ground tracks of the first 8 GPS satellites in this constellation are shown in Figure 8. These ground tracks repeat at different times of day for the remaining 16 satellites in the 24-satellite constellation. The SVHDOP provided by the CIGNET geometry for observing the satellite along-track and cross-track errors using equation (7) is illustrated in Figure 9 for satellites 0 through 7. Because of the symmetry in the constellation, the pattern repeats for satellites 8 through 23. The periods of visibility for the GPS satellites over the United States are also illustrated in Figure 9.

An extended CIGNET network of stations is illustrated in Figure 10. It includes additional stations at Portland, Denver, Houston, and Anchorage in the United States, and at Gander (Newfoundland), Churchill (Canada), Bermuda, and Acores. This provides the improved satellite observation geometry illustrated in Figure 11.

Equation (9) shows that an SVHDOP of less than 10 is required before the differential network can improve on the accuracy of the GPS satellite ephemeris

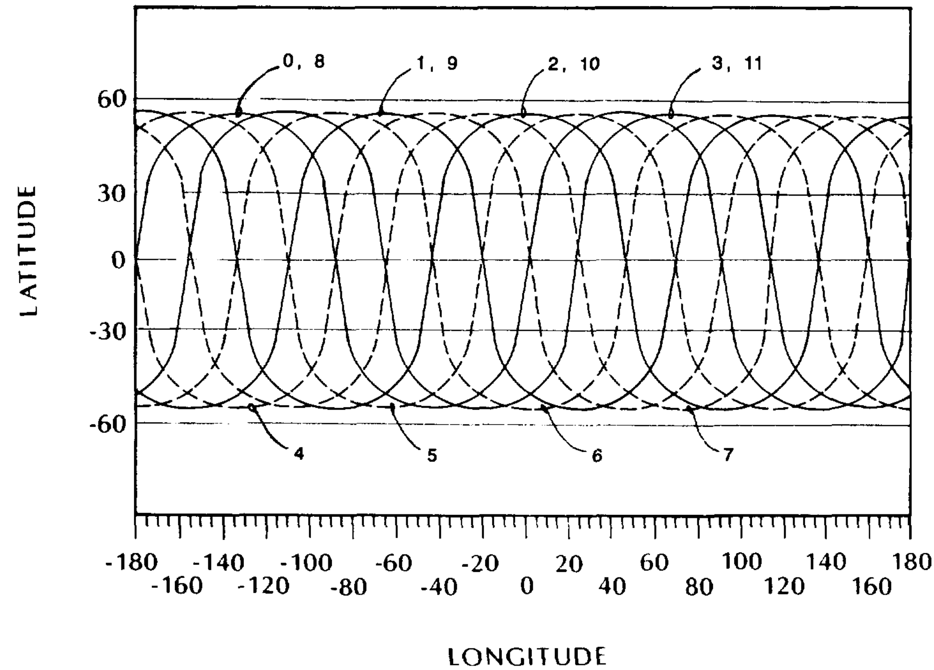

Fig. 8—24-Satellite Constellation Ground Tracks

data. Figures 9 and 11 show that both the CIGNET and extended CIGNET networks provide SVHDOPs as small as 4 for periods of time. The extended CIGNET network provides this geometry for a larger percentage of time when the satellites are visible over CONUS. With an SVHDOP of 4, the extended differential network can broadcast ATK, XTK, RAD, and time/SA corrections that will reduce the satellite ephemeris errors to 1.2 ft over a 1000 nmi range. If additional stations are included in the network, particularly on the Asian continent, it should be possible to estimate the satellite ephemeris errors accurately before the satellites are visible over the United States. The simple least squares solution shown in equation (7) makes it very easy to process data from large numbers of stations all over the world.

Extended Differential Navigation Accuracy

The navigation accuracy that can be achieved using the extended differential GPS method is summarized in Table 4.

It was shown earlier that the satellite ephemeris, clock, and SA errors can be estimated using an inverse ranging technique from the different reference stations in the differential network. A separate correction can then be broadcast for each satellite for the combined clock/SA/radial satellite errors and for the along-track and cross-track satellite position errors. This reduces the range error contribution from the satellite along-track and cross-track errors to below 3 ft over a ± 1000 nmi range. The residual clock, SA, and radial position error introduces a 1 ft range error due to the difficulty in resolving between these different error sources.

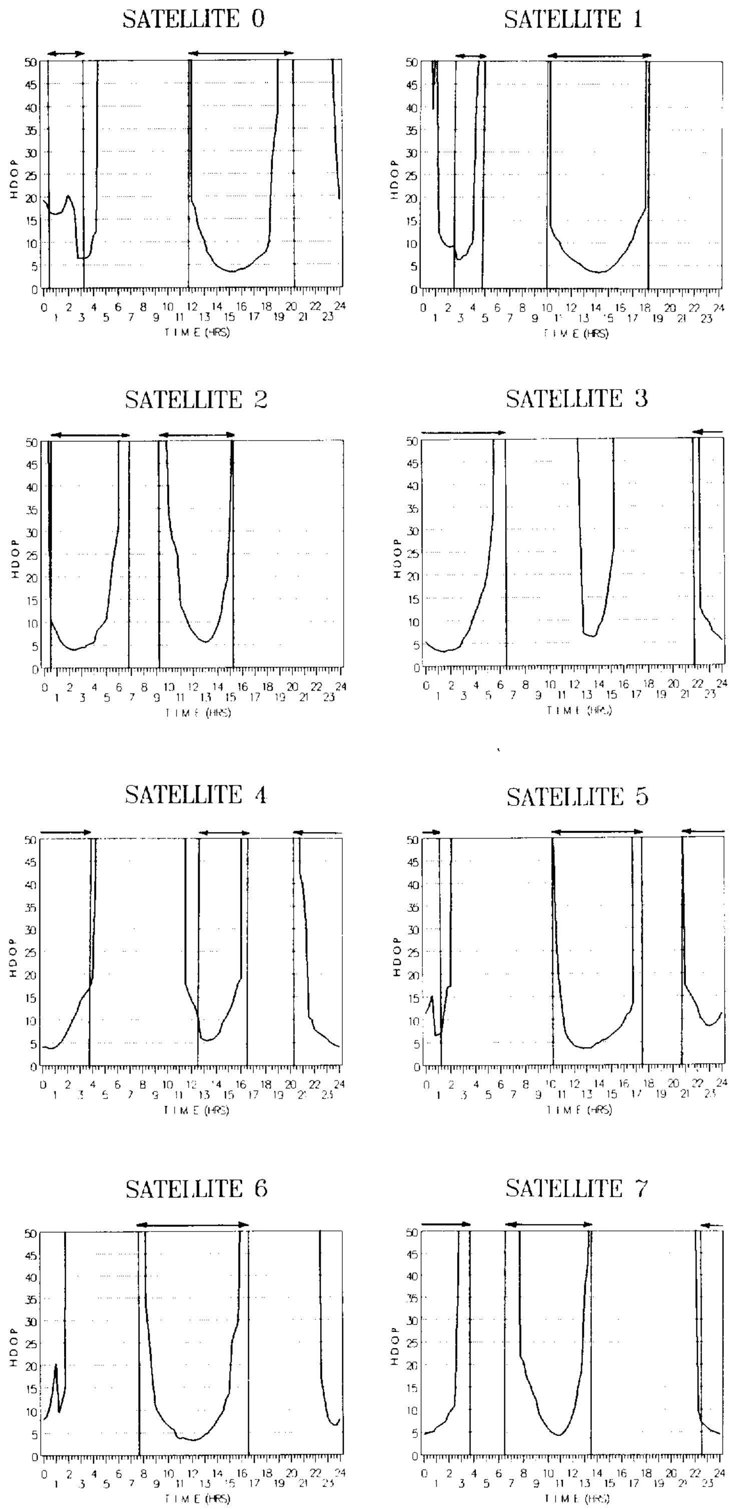

Fig. 9—CIGNET Network Geometry

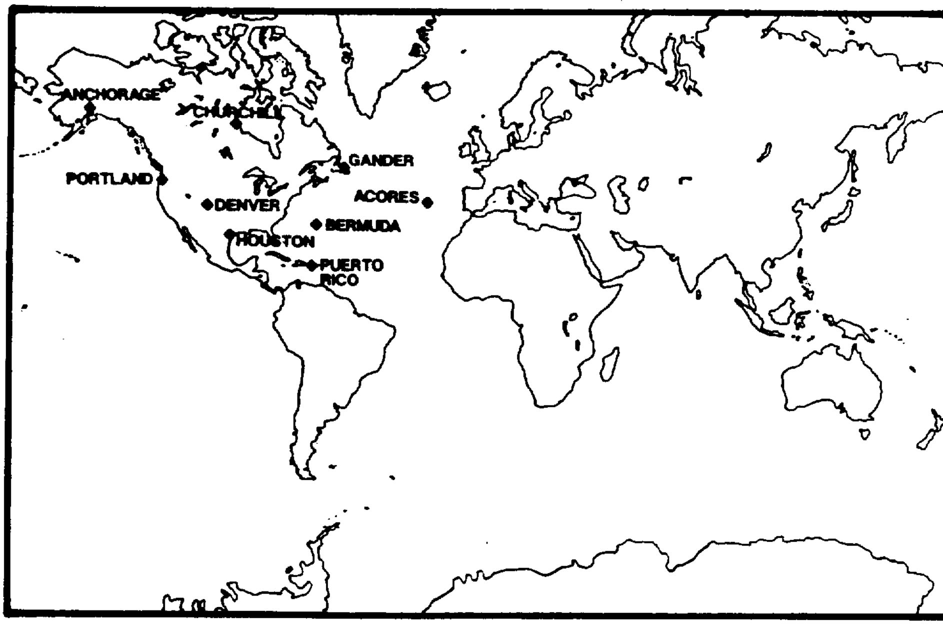

Fig. 10—Additional Differential Stations

The extended differential GPS message must also correct for atmospheric effects. As described earlier, an updated eight-coefficient GPS ionospheric model can be computed from dual-frequency GPS measurements throughout the extended differential GPS network. Using this model instead of the model broadcast by the GPS satellites will reduce the residual ionospheric delay to below 6 ft over a ±1000 nmi range. The residual tropospheric delay can also be reduced by broadcasting an improved model. A three-coefficient updated AFCRL tropospheric model should reduce the residual error to below 1 ft over the extended range.

The additional message types that are required for the extended differential GPS broadcast are listed in Table 5. RTCM SC-104 has tentatively allocated message types 15 and 17 for the broadcast of ionospheric, tropospheric, and satellite ephemeris data. The suggested content for these message types is shown in Table 5.

Message type 15 is reserved in the SC-104 format for ionospheric and tropospheric models. Using the models described earlier, this message would contain the updated eight-coefficient GPS ionospheric model and an updated three-coefficient tropospheric model. Message type 17 is tentatively reserved for satellite ephemeris data. Instead of broadcasting the complete satellite ephemeris data, it is suggested that only an along-track and cross-track error be included for each satellite with the time of applicability (t_{oe}). This error should then be applied as a correction at the receiver to the broadcast satellite ephemeris data. This approach significantly reduces the size of the message while still providing accurate corrections.

The pseudorange corrections broadcast in message type 1 will include the combined satellite clock/SA/radial errors computed by the differential GPS

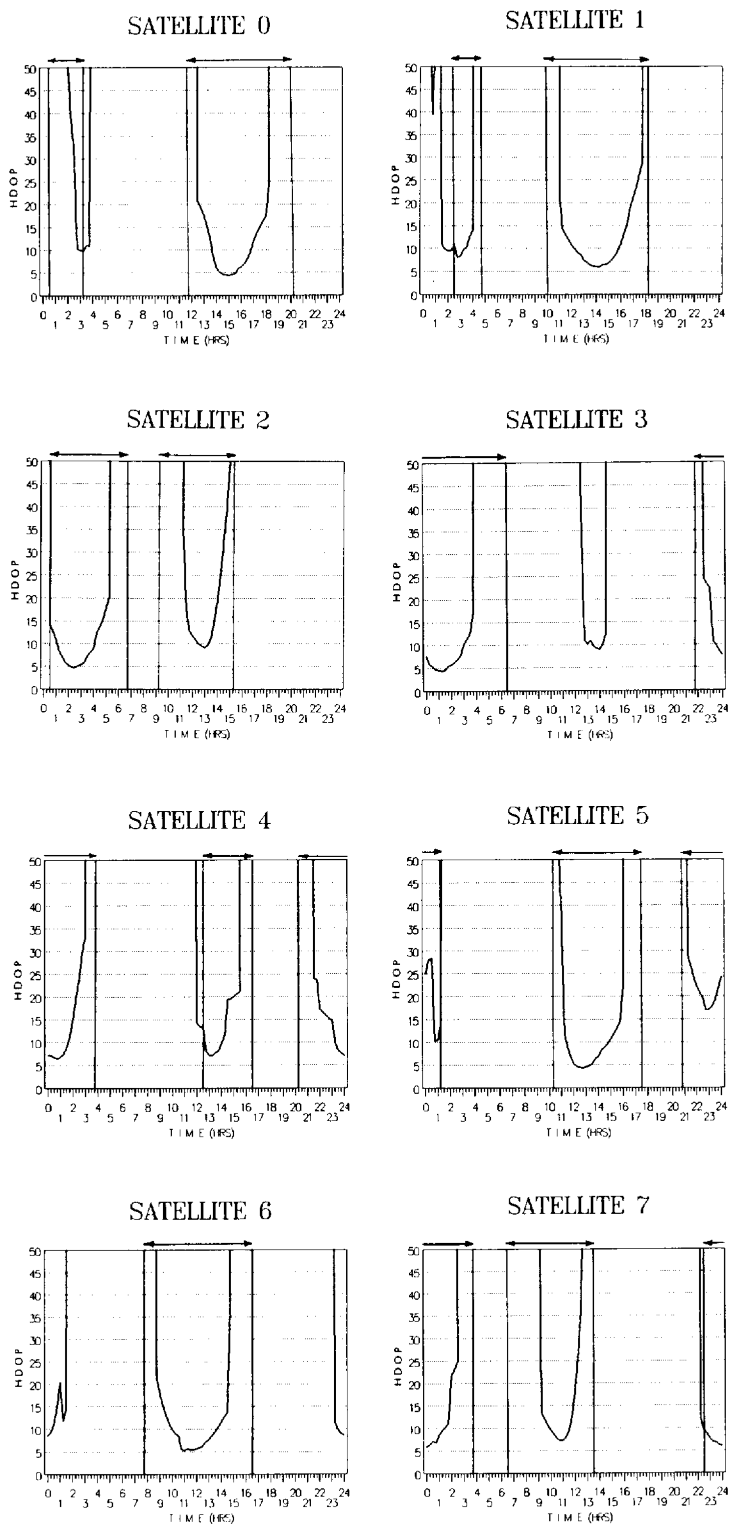

Fig. 11—Extended CIGNET Network Geometry

Table 4—Extended Differential Navigation Accuracy

Source	Error Budget 2000 nmi range (ft)
Space Segment	
Clock Error	0
Control Segment	
Ephemeris Error[1]	3
Selective Availability[2]	1
Atmospheric Effects	
Ionospheric[3]	6
Tropospheric[4]	1
Total (RMS)	7.4
User Segment	
Receiver noise[5]	3
Multipath[6]	0
UERE (RMS)	8
Navigation Accuracy	
2 dRMS (HDOP = 1.5)	24

Notes
[1]Residual ATK and XTK errors from ephemeris estimates.
[2]Residual radial/clock/SA ephemeris errors estimation.
[3]Residual error in updated eight-coefficient model.
[4]Residual error in updated tropospheric model.
[5]Receiver noise is filtered.
[6]Multipath rejection antenna used.

Table 5—Extended Differential Messages

Message Type	Data Content
1—Range errors	SV clock, SA, and radial position errors
15—Iono/Tropo	8—coefficient ionospheric model 3—coefficient tropospheric model
17—Ephemeris	ATK and XTK errors at t_{oe} for each satellite

master station for each satellite. This is the only message that needs to be broadcast by the extended differential service at a high rate since the SA errors change with time.

SUMMARY

The conventional differential navigation error budget is shown in Table 2. Conventional differential GPS using Radio Technical Commission for Maritime Services Special Committee (SC)-104 message type 1 provides a navigation accuracy of 30 ft (2 dRMS) over a range of 100 nmi from the reference station. With the Extended Differential GPS concept, separate corrections are broadcast in the message for the clock and selective availability (SA), ephemeris, ionospheric, and tropospheric errors on each satellite's signal. This allows the differential GPS service to be extended over a range of ±1000 nmi, providing a navigation accuracy of 24 ft (2 dRMS) or better.

Figure 1 illustrates an extended differential GPS network operating over CONUS. Each individual differential station in the network provides the satellite range error corrections to a master differential station. The master station uses these and other measurements from the reference stations to compute an extended differential GPS message that separates out the different error sources. This message is then broadcast to users over a 1000 nmi region using a satellite communications link. By using two differential master stations and satellite links, an extended differential GPS service could be provided that would cover the entire continental United States.

Differential GPS has already proven to be a valuable service for many applications. Once SA has been implemented and the accuracy of the Standard Positioning Service has been degraded to 100 m (2 dRMS), more applications will require differential GPS corrections. The Extended Differential GPS method will allow an economical, nationwide, differential GPS service to be implemented which will provide high-accuracy navigation to a large number of users.

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