Mathematical Aspects of GPS RAIM

Two Results on Integrity Monitoring and Fault Detection

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Two new theorems will answer two questions about RAIM and the least squares navigation solution.

(1) What can be achieved by RAIM from a snapshot of data?

(2) What is the relationship between the two nav solutions obtained by

(a) excluding a biased measurement ("active" RAIM)

(b) including the biased measurement but compensating for its effect using RAIM techniques ("passive" RAIM)

Background
Least Squares Navigation Solution
Error & Fault Vector

Standard measurement equation:

$$z = Hx + e$$

Least squares solution:

$$\hat{x} = H^{*}z$$

where $H^{*} = (H^{T}H)^{-1}H^{T}$

Navigation error:

$$\begin{aligned} d &:= \hat{x} - x \\ &= H^{*}z - x \\ &= H^{*}(Hx + e) - x \\ &= H^{*}e \end{aligned}$$

Fault vector:

$$f := Sz$$

where $S := I - HH^{*}$

$$\begin{aligned} f &:= S(Hx + e) \\ &= Se \end{aligned}$$

Theorem 1: Independence of f and d

Question:

Given a snapshot of measurements, is it ever possible to determine the navigation error from the fault vector?

Answer:

No.

Theorem:

Consider the measurement equation, f, and d.

$$z = Hx + e$$
$$f := Sz$$
$$d := \hat{x} - x = H^{*} e$$

If the elements of e are IID random variables, then f is statistically independent of d.

Theorem:

e IID $\Rightarrow$ *f* is independent of *d*.

Proof:

svd of H:

$$H = U\begin{bmatrix}\Sigma \\ 0\end{bmatrix}V^T$$

$$H^* = V[\Sigma^{-1}\ 0]U^T$$

$$\begin{aligned} S &= I - HH^* \\ &= U\begin{bmatrix}0 \\ I\end{bmatrix}[0\ I]U^T \end{aligned}$$

$$\begin{aligned} f &= Se \\ &= U\begin{bmatrix}0 \\ I\end{bmatrix}[0\ I]U^T e \end{aligned}$$

$$\begin{aligned} d &= H^* e \\ &= V[\Sigma^{-1}\ 0]U^T e \end{aligned}$$

$$\tilde{e} := \begin{bmatrix}\tilde{e}_1 \\ \tilde{e}_2\end{bmatrix} := U^T e$$

$$f = U\begin{bmatrix}0 \\ I\end{bmatrix}\tilde{e}_2, \quad d = V\Sigma^{-1}\tilde{e}_1$$

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Background Math, Active RAIM
Fault (Parity) Vector & Error Estimate

Active RAIM; measurement equation:

$$z_{0i} = H_{0i}x + e_{0i}$$

Fault vector:

$$f = Se$$

$$f_i = S_{i\bullet}e$$

Error model:

$$\hat{e} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \hat{e}_i \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Bias estimate:

$$\hat{e}_i = \frac{f_i}{S_{ii}}$$

Nav error estimate:

$$\begin{aligned} \hat{d} &= H^{*}\hat{e} \\ &= H^{*}_{\bullet i}\,\hat{e}_i \\ &= H^{*}_{\bullet i}\,\frac{S_{i\bullet}}{S_{ii}}e \end{aligned}$$

Theorem 2 : Equivalence of active & passive RAIM

Consider the three navigation solutions

$\hat{x} := H^* z$	biased nav solution
$\hat{x}_{RAIM} := \hat{x} - \hat{d}$	compensated solution (passive RAIM)
$\hat{x}_{0i} := H_{0i}^* z_{0i}$	unbiased solution (active RAIM)

Question:

What is the relationship between $\hat{x}_{RAIM}$ and $\hat{x}_{0_i}$?

Answer:

They are identical.

Theorem:

$$\hat{x}_{RAIM} = \hat{x}_{0i}$$

Proof needs a lemma and its corollary.

Lemma:

$$H_{0i}^* = H^* - H^*_{\bullet i} \frac{S_{i\bullet}}{S_{ii}}$$

Corollary:

The ith column of H_{0i}^* equals zero.

Theorem:

$$\hat{x}_{RAIM} = \hat{x}_{0i}$$

Proof:

$$\begin{aligned}
\hat{x}_{RAIM} &:= \hat{x} - \hat{d} \\
&= H^* z - H^*_{\bullet i} \frac{S_{i\bullet}}{S_{ii}} e \\
&= H^* (Hx + e) - H^*_{\bullet i} \frac{S_{i\bullet}}{S_{ii}} e \\
&= x + (H^* - H^*_{\bullet i} \frac{S_{i\bullet}}{S_{ii}}) e \\
\hat{x}_{0i} &= H_{0i}^* z_{0i} \\
&= H_{0i}^* (H_{0i} x + e_{0i}) \\
&= x + H_{0i}^* e_{0i} \\
&= x + H_{0i}^* e
\end{aligned}$$

Recall Lemma:

$$H_{0i}^* = H^* - H^*_{\bullet i} \frac{S_{i\bullet}}{S_{ii}}$$

Using the lemma, the coefficients of e are equal.

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Example
(from the GPS Toolbox for Matlab)

» help raimdemo

raimdemo

function to run lsnav.m with some real data collected at Wallops Island, VA, USA

The user has the option of corrupting one of the pseudo-ranges with a bias,

raim.m is then run to try and detect the bias and its effect on the navigation solution.

This demo illustrates the use of raim.m with lsnav.m

» raimdemo

There are 8 satellite measurements, from satellites:
2, 7, 13, 14, 15, 19, 27, 31

With elevations (in degrees): 71, 49, 21, 20, 41, 16, 37, 5

Select a satellite to bias :2

Select the bias in meters :100

A bias of 100m has been added to satellite 2

Navigating, using only the 7 unbiased satellites

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Navigating using all 8 satellites, with RAIM to detect bias and correct navigation error

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Hit any key to continue

Correcting the biased solution with the RAIM estimate of the error

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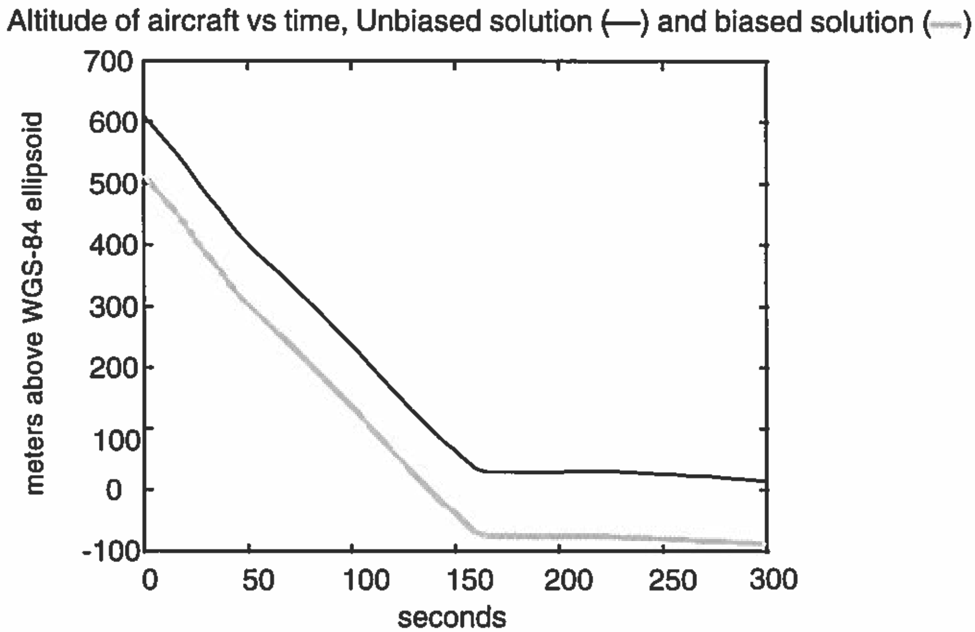
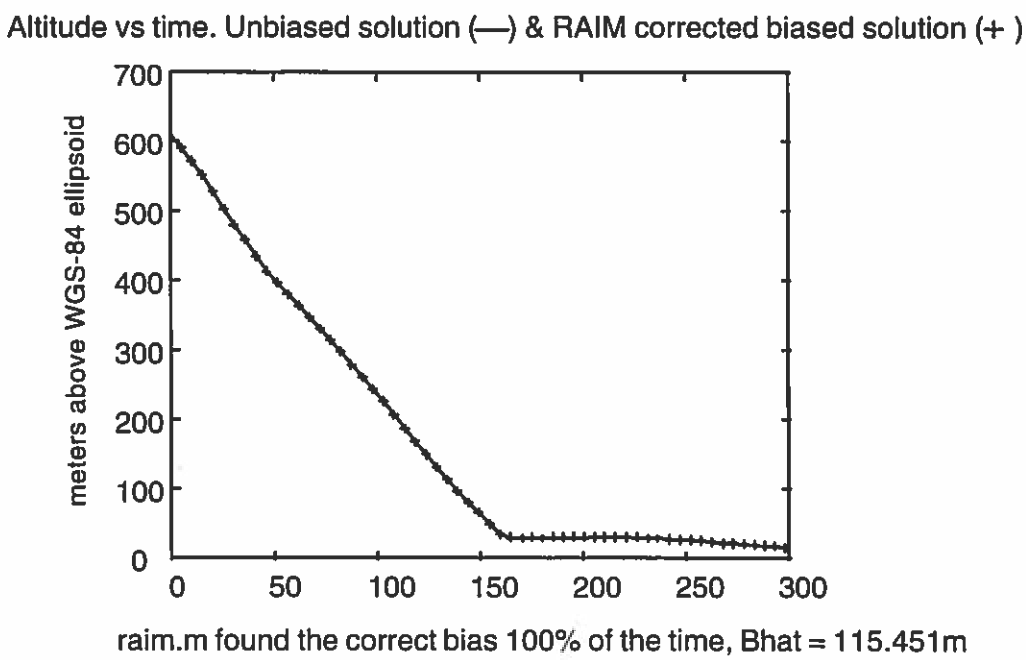
Example
(from the GPS Toolbox for Matlab)



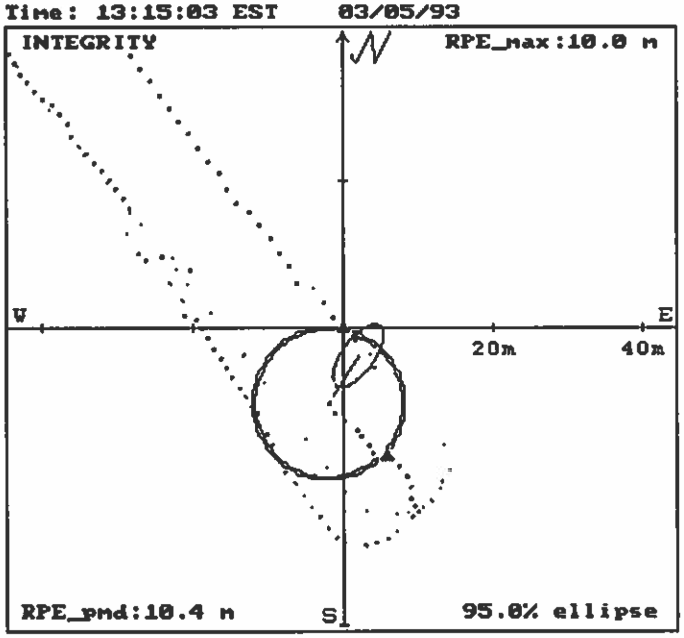


Example
(from USCG R&D Field Test)

POSITION: 41°19'06.572" N 72°03'57.701" W 3.02 m

Rx Status: DGPS 3D #SATS: 07 RPE_est: 7.69 m RPE_thresh: 7.32 m

Time: 13:15:03 EST 03/05/93



ALARMS — % Occurrence since: 13:01:47 EST 03/05/93

Red BIAS RPE > RPE_max PFA < PFA_max	2.05 %
Yellow PFA > PFA_max RPE > RPE_max	0.12 %
Green RPE < RPE_max PMD < PMD_max	97.82 %

Geometric Parameters

Sigma_pr:	1.463 m
HDOP:	1.02
HDOPi:	1.50
Delta_Hi:	1.10
Delta_Hreq:	2.55

SV: 3 25 19 11 28 18 29

Bias Est (m): 13.65

PMD_max: 5% PFA_max: 5% History: -1

	F1	F2	F3	F4	F5	F6	F7	F8	F9	F10	+/-
	Switch	Log	Disp	History	RPE	PFA	PMD	Ellipse	Occur	Scale	Scale
ALT	NavPlan	Mag	Mask	WayPt	Refresh	σ_pr	MDFA				

Conclusion

Theorem 1:

It is essential to take an ensemble of measurements to predict the precision of the nav solution.

Theorem 2:

If a biased measurement is detected, it is just as good to correct the biased nav solution as to exclude the measurement before making the nav solution.